M.Sc. DEGREE EXAMINATION, NOV. - 2015

THIRD SEMESTER

Branch : Mathematics

MA 301 : COMMUTATIVE ALGEBRA

(Revised Syllabus w.e.f. 2015 -16)

(Common for both CBCS & Non - CBCS)

Time : 3 Hours Max. Marks :90

SECTION - A

Answer any **Four** of the following questions. **All** questions carry **equal** Marks

(Marks : 4×4½ = 18)

1. If N is a submodule of an R - module M, then prove that the group of cosets $x+N$ can be made into an R - module M , which is an R - homomorphic image of M with kernel N. **(4½)**

2. The radical of u is an ideal containing u. Then prove that

$$\sqrt{u\,B} = \sqrt{u \cap B} = \sqrt{u} \cap \sqrt{B}$$ **(4½)**

3. If L and N are submodules of M, then show that $l(L)+l(N)=l(L+N)+l(L\cap N)$. **(4½)**

4. State and Prove Holder theorem. **(4½)**

5. Prove that every non - unit 'a' in a noetherian domain R is a product of irreducible elements. **(4½)**

6. Prove that in a ring R having only one maximal ideal m , every idempotent e is either 0 (or) 1. **(4½)**

7. Prove that in a ring with a.c.c. every irreducible ideal is primary. (4½)

8. Let R be a noetherian ring and a and m two ideals of R. Then prove that $\exists$ an integer S and an ideal a' of R such that $ma = ana'$ and $a' \supset m^s$ (4½)

SECTION - B

Answer **All** questions. **Each** question carries **18** Marks

(Marks : 4×18= 72)

9. a) State and prove isomorphism theorems. (18)

OR

b) i) Let u be an ideal different from R. Then prove that if R has an identity u is maximal if and only if R/u is a field. (18)

ii) Let D be a primary ideal in a ring R. If B is the radical of D, then prove that B is a prime ideal. Moreover, if $ab \in D, and\ a \notin D, then\ p.t\ b \in B$

10. a) i) Prove that a module M satisfies the ascending (descending chain condition if and only if satisfies the Maximum (Minimum) condition. (18)

ii) State and prove Jordan theorem.

OR

b) i) Let M be a module over a ring R. Then prove that M satisfies the ascending chain condition if and only if every submodule of M has a finite basis. (18)

ii) Let $M_1', \ldots, M_r'$ be modules over a ring R. Then prove that there exists a module M which is the direct sum of submodules $M_1, \ldots, M_r$ such that M_i is R - isomorphic to M_i'. Moreover, M is uniquely determined up to R - isomorphism.

11. a) Prove that R is a noetherian ring, then so is any polynomial ring in a finite number of indeterminates over R. (18)

(OR)

b) Prove that A ring with identity R which satisfies the descending chain condition is the direct sum of noetherian primary rings, and this decomposition is unique. **(18)**

12. a) i) Let R be a ring and let 'a' be an ideal of R admitting an irredundant primary representation $a = n_i a_i$. Then prove that for 'a' to be its own radical, it is necessary and sufficient that all a_i be prime ideals. **(18)**

ii) Let R be a noetherian ring and, a and b two ideals of R such that $a \neq R$. Then prove that a = a:b if and only if b is contained in no prime ideal of 'a'

(OR)

b) i) Let R be an arbitrary ring with identity, and let 'a' and u are two ideals of R such that 'a' admits a finite basis $(x_1, \ldots x_n)$ and $a = am$. Then prove that there exists an element Z in m such that $(1 - Z)a = (0)$. **(18)**

ii) Let R be a noetherian ring and m an ideal of R. Then prove that in order that $\bigcap_{n=1}^{\infty} m^n = (0)$, It is necessary and sufficient that no element of 1 - m be a zero - divisor in R.

236-03-05A -R

M.Sc. DEGREE EXAMINATION, NOV.- 2015
THIRD SEMESTER
Branch : Mathematics
PAPER V(A) - CLASSICAL MECHANICS

(CBCS w.e.f 2015-16)

(Common to Applied Mathematics)

(Common to suppl.can. also ie, who failed in Nov. 2014 and earlier with 100 marks)

Time : 3 Hours Max. Marks : 90

PART - A

Answer any FOUR questions. Each question carries 4½ marks.

(Marks : 4×4½=18)

1. Write mathematical expression for the principle of virtual work.

2. Prove that in a simple dynamical system T+V= constant where T is kinetic energy and V is potential energy.

3. Derive Hamilton's principle from Lagrange's equations.

4. Find the curve for which the surface of resolution is minimum.

5. If the equation of transformation do not depend explicitly on time and if the P.E. is velocity dependent then H is the total energy of the system.

6. Prove that $\frac{dH}{dt} = \frac{\partial H}{\partial t}$.

7. Show that (a) [u, u] = 0, (b) [u+v, w] = [u, w]+[v, w]

8. Explain canonical coordinates and canonical transformation.

PART - B

Answer One question from each unit. Each question carries 18 marks.

(Marks : 4×18=72)

UNIT - I

9. Derive Lagrange's equations for a Holonomic system.

(OR)

10. a) Explain D-Alembert'z principle.

    b) Write short notes on Harmonic Oscillator.

UNIT - II

11. Use Hamilton's principles to find the equations of motion of a particle of unit mass moving on a plane in a conservative field.

(OR)

12. a) State and prove conservation theorem of Linear momentum.

    b) Derive conservation theorem.

UNIT - III

13. State and prove Brachistrone problem.

(OR)

14. Derive Hamilton's equation of motion from Hamilton's principle.

UNIT - IV

15. State and prove Jacobi's identity for Poisson brackets.

(OR)

16. Show that the Hamiltons equations of motion in poisson bracket are special cases of general formula for the total time derivative of a function u(q,p,t).

M.Sc. DEGREE EXAMINATION, NOV. - 2015

THIRD SEMESTER

BRANCH : MATHEMATICS

MA 304(A) : DIFFERENTIAL GEOMETRY

(Revised Syllabus w.e.f. 2015 -16)(Common for both CBCS & Non - CBCS)

(Common with paper AM 304(A) of Branch I(B) Applied Mathematics)

Time : 3 Hours

Max. Marks : 90

SECTION - A

Answer any **Four** of the following questions. All questions carry **equal** Marks

(Marks : 4×4½ = 18)

1. Derive a formula for the arc length of a curve **(4½)**

2. Show that the involutes of a circular helin are plane curves **(4½)**

3. Find a surface of revolution which is isometric with a region of the right helicoid **(4½)**

4. Prove that, on the general surface, a necessary and Sufficient condition that the curve

   V = c be a geodesic is $EE_2 + FE_1 - 2EF_1 = 0$ when V = c for all values of u **(4½)**

5. Show that Every point of a surface has a neighbourhood which is convex and simple. **(4½)**

6. Prove that if A is the area of a geodesic disk of centre p and radius r, then

   $$(K)_p = \lim_{r \to 0} \frac{\pi r^2 - A}{\frac{1}{12}\pi r^4}$$ **(4½)**

7. If there is a surface of minimum area passing through a closed space curve then prove that it is minimal. **(4½)**

8. Show that the meridians and parallels of a surface of revolution are its lines of curvature. **(4½)**

SECTION - B

Answer **All** questions. Each question carries **18** Marks

(Marks : 4×18= 72)

9. a) i) State and prove fundamental Existence theorem for space curves **9**

ii) Calculate the curvature and Torsion of the curve given by $r = \{a(1-\sin u), a(1-\cos u), bu\}$ **9**

(OR)

b) i) Let 'γ' be a curve for which 'b' varies differentiably with arc length. Then prove that the necessary and sufficient condition that 'γ' be a plane curve is that $\tau = 0$ at all points **9**

ii) Show that the spherical indicatrix of a curve is a circle if and only if the curve is a helix **9**

10. a) i) Find the orthogonal trajectories of the sections by the planes z = constant on the paraboloid $x^2 - y^2 = z$ **9**

ii) Derive differential Equation for geodesics **9**

(OR)

b) If g(t) is continuous for $0 < t < 1$ and if $\int_0^1 V(t)g(t)dt = 0$ for all admissible functions V(t), then show that g(t) = 0 **18**

11. a) i) Prove that Every helix on a cylinder is a geodesic **9**

ii) Prove that if (λ, μ) is the geodesic curvature vector, then

$$K_g = -\frac{-H\lambda}{Fu^1 + Gv^1} = \frac{H\mu}{Eu^1 + Fv^1}$$ **9**

(OR)

b) State and prove Minding's theorem. **18**

12. a) i) Derive Rodrigue's formula for $Kdr + dN = 0$ **9**

ii) Prove that the necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature shall be zero. **9**

(OR)

b) i) Show that the surfaces $e^x \cos x = \cos y$ is minimal. **9**

ii) Prove that the Gaussian curvature is the same at two points of a generator which are equidistant from the central point. **9**

[Total No. of Pages : 2

A-236-03-03

M.Sc. DEGREE EXAMINATION, JANUARY - 2017

THIRD SEMESTER

Branch : MATHEMATICS / APPLIED MATHEMATICS

PAPER - III : DISCRETE MATHEMATICS

(Under C.B.C.S. & Non-CBCS w.e.f. 2016-2017)

(Common to Suppl. candidates also i.e., who appeared in 2015 with 90 marks and earlier with 100 marks.)

Time : 3 Hours Max. Marks : 90

Section - A

Answer any Four questions. All questions carry equal marks.

(Marks: 4×4½=18)

1. Show that $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology.

2. Determine whether the conclusion $C: \neg(P \wedge Q)$ follows logically from the premises $H_1: \neg P$ and H_2 $P \rightleftarrows Q$.

3. Symbolize the predicate "x is the father of the mother of y".

4. Negate the statement "Every city in Canada is clean".

5. Draw the diagram of lattice (S_{30}, D).

6. Show that, in a distributive lattice $<L, *, \oplus>$, for any a, b, c $\in$ L, $(a * b = a * c) \wedge (a \oplus b = a \oplus c) \Rightarrow b = c$.

7. If $S_i \equiv S_j$, then show that $\delta(S_i, x) \equiv \delta(S_j, x)$ for any input sequence x.

8. Define adjacency matrix, path matrix of a graph with examples

Section - B

Answer **All** questions. All questions carry **equal** marks **(4×18=72)**

9. i) Obtain the product-of-sums canonical form of the formula

$(p \wedge Q \wedge R) \vee (\neg p \wedge R \wedge Q) \vee (\neg p \wedge \neg Q \wedge \neg R)$

ii) Obtain the principal conjunctive normal form of $(p \to (Q \wedge R)) \wedge (\neg p \to (\neg Q \wedge \neg R))$

OR

10. i) Show that SVR is tautologically implied by $(p \vee Q) \wedge (p \to R) \wedge (Q \to S)$

ii) Show that $\overset{S}{\Rightarrow}\left(\neg Q \wedge (P \rightarrow Q)\right) \rightarrow \neg P$

11. i) Indicate free and bound variables in the expression $(x)(P(x) \wedge R(x)) \rightarrow (x)P(x) \wedge Q(x)$

ii) Find the truth values of $(\exists x)(P(x)) \rightarrow Q(x) \wedge T$, Where $P(x): x > 2, Q(x): x = 0$ *and* T is any tautology with the universe of discourse as $\{1\}$.

OR

12. i) Show that $(\exists x)M(x)$ follows logically from the premises $(x)(H(x) \rightarrow M(x))$ *and* $(\exists x)H(x)$

ii) Show the following by constructing derivations $(x)(P(x) \rightarrow (Q(y) \wedge R(x))), (\exists x)P(x) \Rightarrow Q(y) \wedge (\exists x)(P(x) \wedge R(x))$.

13. i) In a lattice, show that $a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$ and $a * (b \oplus c) \geq (a * b) \oplus (a * c)$

ii) Show that Every chain is a distributive lattice.

OR

14. i) Show that $[a * (b' \oplus c)]' * \left[b' \oplus (a * c')'\right]' = a * b * c'$.

ii) Use the karnaugh map representation to find a minimal sum - of - products expression of the function $f(a,b,c,d) = \sum(0,5,7,8,12,14)$.

15. i) Let S be any state in a finite state machine and x, y be any words. Then show that $\delta(S,xy) = \delta(\delta(S,x),y)$ *and* $\lambda(S,xy) = \lambda(\delta(S,x),y)$

ii) Show that, In a simple graph, the length of any elementary path is less than or equal to n - 1.

OR

16. i) Let $S_i, S_j \in S$. Show that $S_i \overset{k+1}{\equiv} S_j$ if and only if $S_i \overset{k}{=} S_j$ and for all $a \in I$, $\delta(S_i, a) \overset{k}{\equiv} \delta(S_j, a)$

ii) Show that in a complete binary tree, the total number of edges is $2(n_t - 1)$ where n_t is the number of terminal nodes

236-03-02 R

M.Sc. DEGREE EXAMINATION, NOV. - 2015
THIRD SEMESTER
Branch - MATHEMATICS
Paper II - FUNCTIONAL ANALYSIS

(Common to Applied mathematics)

(w.e.f. 2015-16)

(Common for both CBCS and Non-CBCS)

(Common to suppl. can. also ie, who appeared in Nov-2014 and earlier)

Time : 3 Hours

Max. Marks : 90

PART - A

Answer any **FOUR** of the following questions. Each question carries **4½** marks.

(Marks : 4×4½=18)

1. Prove Holder inequality

2. If M is a closed linear subspace of a normed linear space N and x_0 is a vector not in M, then prove that there exists a functional f_0 in N*, such that $f_0(M) = 0$ and $f_0(x_0) \neq 0$.

3. Show that one-to-one continuous linear transformation of one Banach space onto another is a homeomorphism.

4. If B and B\` are Banach spaces, and if T is a linear transformation of B into B\`, and T is continuous then show that its graph is closed.

5. Show that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

6. State and prove parallelogram Law.

7. If A_1 and A_2 are self-adjoint operators on H, then their product A_1A_2 is self - adjoint $\Leftrightarrow A_1A_2=A_2A_1$

8. If N is a normal operator on H, then show that $\|N^2\| = \|N\|^2$

PART - B

Answer **ONE** question from each unit. Each question carries **18** marks.

(Marks : 4×18=72)

UNIT - I

9. Let N and N' be normed linear spaces and T a linear transformation of N into N'. then the following conditions on T are all equivalent to one another :

i) T is continuous ;

ii) T is continuous at the origin, in the sense that $x_n \to 0 \Rightarrow T(x_n) \to 0$;

iii) There exists a real number $K \geq 0$ with the property that $\|T(x)\| \leq K\|x\|$ *for every* $x \varepsilon N$

iv) If $S = \{x : \|x\| \leq 1\}$ is the closed unit sphere in N, then its image T(S) is bounded set in N'.

(OR)

10. If N and N' are normed linear spaces, then the set B(N, N') of all continuous linear transformations of N into N' is itself is a normal linear space with respect to the pointwise linear operations and the norm defined by $\|T\| = \sup\{\|T(x)\| : \|x\| < 1\}$. Further, if N' is a Banach space, then B(N, N') is also a Banach space.

UNIT - II

11. State and prove open mapping theorem.

(OR)

12. a) If P is a projection on a Banach space B and if M and N are its range and null space then prove that M and N are closed linear sub-spaces of B such that B=M $\oplus$ N.

b) State and prove uniform boundedness theorem.

UNIT - III

13. Let H be a Hilbert space, and let $\{e_i\}$ be an orthonormal set in H. Then the following conditions are all equivalent to one another :

i) $\{e_i\}$ is complete;

ii) $x \perp \{e_i\} \Rightarrow x = 0$;

iii) If x is an arbitrary vector in H, then $x = \Sigma(x, e_i)e_i$;

iv) If x is an arbitrary vector in H, then $\|x\|^2 = \Sigma|(x, e_i)|^2$

(OR)

14. a) Let $\{e_1, e_2, ..., e_n\}$ be a finite orthonormal set in a Hilbert space H. if x is any vector in H, then prove that $\sum_{i=1}^{n}|(x, e_i)|^2 \leq \|x\|^2$; further, $x - \sum_{i=1}^{n}(x, e_i)e_i \perp e_j$ *for each j*

b) State and prove BESSEL inequality.

UNIT - IV

15. If $P_1, P_2, ..., P_n$ are the projections on closed linear sub-spaces $M_1, M_2, ...,M_n$ of H, then $P=P_1+P_2+.....+P_n$ is a projection $\Leftrightarrow$ the P_i' s are pairwise orthogonal (in the sense that $P_iP_j=0$ whenever $i \neq j$); and in this case, P is the projection on $M=M_1+M_2+.....+M_n$.

(OR)

16. State and Prove SPECTRAL Theorem.

[Total No. of Pages : 3

A-236-03-01

M.Sc. DEGREE EXAMINATION, JANUARY- 2017

THIRD SEMESTER

Branch : MATHEMATICS

Paper I : COMMUTATIVE ALGEBRA

(Under CBCS & Non-CBCS w.e.f. 2016-2017)

(Common to Supplementary Candidates also i.e. who appeared in 2015 with 90 marks earlier with 100 marks)

Time : 3 Hours

Max. Marks : 90

SECTION-A

Answer any four questions. All questions carry equal marks. **(4×4½=18)**

1. If N and L are submodules of an R-module M, then show that $(L+N)-N \cong L-(L \cap N)$.

2. Define Prime and Maximal ideals with examples.

3. Let M be a module and N be a submodule. Then prove that the ascending (descending) chain condition holds in M if and only if it holds in both N and M-N.

4. If a completely reducible module satisfying chain condition, then prove that it satisfies the other, and hence has finite length.

5. In a noetherian ring R, prove that every ideal u contains a power of its radical $\sqrt{u}$.

6. Let R be a ring with identity satisfying the d.c.c. Then prove that the intersection r of all maximal ideals in R is the set of all nilpotent elements of R.

7. In a ring with a.c.c., prove that every irreducible ideal is primary.

8. If an ideal m of a ring R and a family $\{a_\lambda\}$ of ideals of R are closed (with respect to m), then prove that the intersection $\bigcap_\lambda a_\lambda$ is closed.

SECTION-B

Answer all questions. All questions carry equal marks. **(4×18=72)**

Unit - I

9. i) If M' is any R-homomorphism image of M, with kernel N, then prove that the elements of M' are in (1,1) correspondence with those of M-N, and the correspondence is an R-isomorphism.

ii) If L is a subring of a ring R, and N is an ideal in R, then prove that the residue class ring $L/L \cap N$ is isomorphic with the subring (L+N)/N of the residue class ring R/N.

OR

10. i) If R has an identity, then prove that u is maximal if and only if R/u is a field.

ii) Let D and β be ideals in a ring R. Then prove that D is primary and β is its radical if and only if the following conditions are satisfied.

a) $D \subseteq \beta$.

b) If $b \in \beta$, then $b^m \in D$, for some m, (m may depend on b).

c) If $ab \in D, and\ a \notin D, then\ b \in \beta$.

Unit - II

11. i) Prove that a module M satisfies the ascending(descending) chain condition if and only if it satisfies the maximum (minimum) condition.

ii) If an R-module M has one composition series of length r, then prove that every composition series of M has length x, and very normal series without repetitions can be refined to a composition series.

OR

12. Prove that a necessary and sufficient condition that an R-module M be completely reducible and of finite length is that it be the sum of a finite number of simple submodules. Then M is a direct sum of simple submodules, the direct summands are uniquely determined up to R-isomorphism, and their number is l(M).

Unit - III

13. Prove that if R is a noetherian ring, then so is any polynomial ring in a finite number of indeterminates over R.

OR

14. i) Prove that a ring with identity R which satisfies the descending chain condition is the direct sum of noetherian primary rings, and this decomposition is unique.

ii) In a ring R satisfying the d.c.c., then prove that every ideal a, all the elements of which are nilpotent, is nilpotent.

Unit - IV

15. i) Let R be a ring and let a be an ideal of R admitting an irredundant primary representation $a = \bigcap_i q_i$. Then prove that for a to be its own radical, it is necessary and sufficient that all q_i be prime ideals.

ii) Let R be a noetherian ring in which every proper prime ideal is maximal. Then prove that every ideal a of R is, in a unique way, a finite irredundant intersection of primary ideals; a is also, in a unique way, a product of primary ideals belonging to distinct prime ideals.

OR

16. i) Let R be a noetherian ring, **a** and **b** two ideals of R such that $a \neq R$. Then prove that **a** = **a:b** if and only if **b** is contained in no prime ideal of a.

ii) Let R be a noetherian ring and **m** an ideal of R. Prove that, the necessary and sufficient that no element of 1-**m** be a zero-divisor in R is that $\bigcap_{n=1}^{\infty} m^n = (0)$.

[Total No. of Pages : 2 **A-236-03-05(a)**

M.Sc. DEGREE EXAMINATION, JANUARY- 2017

THIRD SEMESTER

Branch : MATHEMATICS / APPLIED MATHEMATICS

Paper V(a) : CLASSICAL MECHANICS

(Under C.B.C.S. & Non - CBCS w.e.f. 2016 - 17)

(Common to supplementary Candidates also i.e., who appeared in 2015 with 90 marks and earlier with 100 marks)

Time : 3 Hours Max. Marks : 90

Section - A

Answer any **Four** questions, All questions carry **equal** marks. **(4×4½=18)**

1. Explain D' Alemberts principle.
2. Explain the use of method of Zagrange's multiplier in holonomic system.
3. Derive Hamilton's principle from Zagrange's equations.
4. Explain conservation of angular momentum
5. If it is not an explicit function of t, then show that it is a constant of the motion.
6. Prove that $\frac{dH}{dt}=\frac{\partial H}{\partial t}$, where it is the Hamiltonian function.
7. Explain canonical transformation and give two examples.
8. If u, v are constants of motion, show that [u, v] is also a constant of motion.

Section - B

Answer **all** questions. All questions carry **equal** marks. **(4×18=72)**

9. Derive Lagrange's equations for a conservative, holonomic, dynamical system.

OR

10. i) Prove that in a simple dynamical system T+V = constant
    ii) Derive the equations of motion for a simple pendulum of length l and mass of bob m by using Lagrange's equations.
11. i) Derive Lagrange's equations from Hamilton's principle.
    ii) Show that the shortest curve between two points in a plane is a straight line.

OR

12. Explain Brachistochrone problems.

13. i) Derive Hamilton's equations of motion using Lagrange's equations.

ii) Explain Routh's procedure

OR

14. Discuss principle of least action.

15. Prove that Poisson's bracket is invariant under canonical transformation.

OR

16. i) Show that the transformation $Q = \log\left(\frac{1}{q}\sin \rho\right), P = q \cot \rho$ is canonical.

ii) For what values of $\alpha and \beta$ do the equations.

$Q = q^{\alpha} \cos \beta\rho, P = q^{\alpha} \sin \beta\rho$

represent a canonical transformation? What is the form of generating function F3 for this case?

[Total No. of Pages : 2

A-236-03-04(a)

M.Sc. DEGREE EXAMINATION, JANUARY- 2017

THIRD SEMESTER

Branch : MATHEMATICS / APPLIED MATHEMATICS

Paper IV(a) : DIFFERENTIAL GEOMETRY

(Under C.B.C.S. & Non - CBCS w.e.f. 2016 - 17)

(Common to supplementary Candidates also i.e., who appeared in 2015 with 90 marks and earlier with 100 marks)

Time : 3 Hours

Max. Marks : 90

Section - A

Answer any **Four** questions, All questions carry **equal** marks. **(4×4½=18)**

1. Find the length of the curve given as the intersection of the surfaces $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, x = a cosh (z/a) from the point (a,0,0) to (x,y,z).
2. Show that the necessary and sufficient condition for a curve to be a plane is $[\dot{r},\ddot{r},\dddot{r}]=0$.
3. Find E, F, G, H for the paraboloid x = u, y = v, z = u^2-v^2.
4. Find the coefficients of the direction which makes an angle $\pi/2$ with the direction whose coefficients are (l,m).
5. If g(t) is continuous for 0<t<1 and if $\int_0^1 v(t)g(t)dt=0$ for all admissible functions v(t) then prove that g(t) = 0.
6. Prove that every point P of a surface has a neighbourhood which is convex and simple.
7. If L,M,N vanish everywhere on a surface then show that the surface is part of a plane.
8. Show that the surface $e^z\cos x=\cos y$ is minimal.

Section - B

Answer **all** questions. All questions carry **equal** marks. **(4×18=72)**

9. Find the curvature and torsion of the curve $ax^2+by^2+cz^2=1, a'x^2+b'y^2+c'z^2=1$

OR

10. Show that the involutes of a circular helix are plane curves.

11. On the paraboloid $x^2 - y^2 = z$ find the orthogonal trajectories of the sections by the planes z = constant.

OR

12. A surface of revolution is defined by the equations $x = \cos u \cos v, y = \cos u \sin v$, $z = -\sin u + \log \tan\left(\frac{\pi}{4} + \frac{u}{2}\right)$ where $0 < u < \frac{\pi}{2}$ and $0 < v < 2\pi$. Show that the metric is $Tan^2 u\, du^2 + \cos^2 u dv^2$. and prove that the region $0 < u < \frac{\pi}{2}, 0 < v < \pi$ is mapped isometrically on the region $\frac{\pi}{3} < u' < \frac{\pi}{2}$, $0 < v' < 2\pi$ by the correspondence $u' = \cos^{-1}\left(\frac{1}{2}\cos u\right), v' = 2v$

13. Find the geodesics on a surface of revolution given by $\bar{r} = (g(u)\cos v, g(u)\sin v, f(u))$

OR

14. State and prove minding theorem.

15. Prove that a necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature shall be zero.

OR

16. The surface of revolution is given by $x = u\cos v, y = u \sin v, z = a \log\left\{u + \sqrt{(u^2 - a^2)}\right\}$ is generated by rotating a catenary about its axis. Prove that it is a minimal surface. Also show that it is the only minimal surface of revolution.

[Total No. of Pages : 2

A-236-03-03

M.Sc. DEGREE EXAMINATION, JANUARY - 2017

THIRD SEMESTER

Branch : MATHEMATICS / APPLIED MATHEMATICS

PAPER - III : DISCRETE MATHEMATICS

(Under C.B.C.S. & Non-CBCS w.e.f. 2016-2017)

(Common to Suppl. candidates also i.e., who appeared in 2015 with 90 marks and earlier with 100 marks.)

Time : 3 Hours

Max. Marks : 90

Section - A

Answer any Four questions. All questions carry equal marks.

(Marks: 4×4½=18)

1. Show that $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology.
2. Determine whether the conclusion $C: \neg(P \wedge Q)$ follows logically from the premises $H_1: \neg P$ and $H_2\ P \rightleftarrows Q$.
3. Symbolize the predicate "x is the father of the mother of y".
4. Negate the statement "Every city in Canada is clean".
5. Draw the diagram of lattice (S_{30}, D).
6. Show that, in a distributive lattice $<L, *, \oplus>$, for any a, b, c $\in$ L, (a * b = a * c) $\wedge (a \oplus b = a \oplus c) \Rightarrow b = c$.
7. If $S_i \equiv S_j$, then show that $\delta(S_i, x) \equiv \delta(S_j, x)$ for any input sequence x.
8. Define adjacency matrix, path matrix of a graph with examples

Section - B

Answer **All** questions. All questions carry **equal** marks **(4×18=72)**

9. i) Obtain the product-of-sums canonical form of the formula $(p \wedge Q \wedge R) \vee (\neg p \wedge R \wedge Q) \vee (\neg p \wedge \neg Q \wedge \neg R)$

   ii) Obtain the principal conjunctive normal form of $(p \to (Q \wedge R)) \wedge (\neg p \to (\neg Q \wedge \neg R))$

OR

10. i) Show that SVR is tautologically implied by $(p \vee Q) \wedge (p \to R) \wedge (Q \to S)$

ii) Show that $\overset{S}{\Rightarrow}(\neg Q \wedge (P \rightarrow Q)) \rightarrow \neg P$

11. i) Indicate free and bound variables in the expression $(x)(P(x) \wedge R(x)) \rightarrow (x)P(x) \wedge Q(x)$

ii) Find the truth values of $(\exists x)(P(x)) \rightarrow Q(x) \wedge T$, Where $P(x): x > 2, Q(x): x = 0$ *and* T is any tautology with the universe of discourse as $\{1\}$.

OR

12. i) Show that $(\exists x)M(x)$ follows logically from the premises $(x)(H(x) \rightarrow M(x))$ *and* $(\exists x)H(x)$

ii) Show the following by constructing derivations $(x)(P(x) \rightarrow (Q(y) \wedge R(x))), (\exists x)P(x) \Rightarrow Q(y) \wedge (\exists x)(P(x) \wedge R(x))$.

13. i) In a lattice, show that $a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$ and $a * (b \oplus c) \geq (a * b) \oplus (a * c)$

ii) Show that Every chain is a distributive lattice.

OR

14. i) Show that $[a * (b' \oplus c)]' * \left[b' \oplus (a * c')'\right]' = a * b * c'$.

ii) Use the karnaugh map representation to find a minimal sum - of - products expression of the function $f(a,b,c,d) = \sum(0,5,7,8,12,14)$.

15. i) Let S be any state in a finite state machine and x, y be any words. Then show that $\delta(S, xy) = \delta(\delta(S,x), y)$ *and* $\lambda(S, xy) = \lambda(\delta(S,x), y)$

ii) Show that, In a simple graph, the length of any elementary path is less than or equal to n - 1.

OR

16. i) Let $S_i, S_j \in S$. Show that $S_i \overset{k+1}{\equiv} S_j$ if and only if $S_i \overset{k}{=} S_j$ and for all $a \in I$, $\delta(S_i, a) \overset{k}{\equiv} \delta(S_j, a)$

ii) Show that in a complete binary tree, the total number of edges is $2(n_t - 1)$ where n_t is the number of terminal nodes

A-236-03-02

M.Sc. DEGREE EXAMINATION, JANUARY - 2017

THIRD SEMESTER

Branch : MATHEMATICS / APPLIED MATHEMATICS

Paper II : FUNCTIONAL ANALYSIS

(Under C.B.C.S. & Non - CBCS w.e.f. 2016 - 17)

(Common to suppl. candidates also i.e., who appeared in 2015 with 90 marks and earlier with 100 marks)

Time : 3 Hours   Max. Marks : 90

Section - A

Answer any **Four** questions. All questions carry **equal** marks. **(4×4½=18)**

1. Define normed linear space and give two examples.
2. If N is a normed linear space and x_0 is a non - zero vector in N, then prove that there exists a functional f_0 in N* such that $f_0(x_0) \equiv \|x_0\|$ and $\|f_0\| = 1$
3. If B is a Banach space and B is reflexive then prove that B* is reflexive.
4. Show that a linear subspace of a normed linear space is closed if it is weakly closed.
5. Let M be a closed linear subspace of a Hilbert space H and x be a vector not in M. Let d be the distance from x to M . Then prove that there exists a unique vector y_0 in M such that $\|x - y_0\| = d$.
6. Let H be a Hilbert space and let $\{e_i\}$ be an orthonormal set in H. Then prove that $\{e_i\}$ is complete if $x \perp \{e_i\} \Rightarrow x = 0$
7. If T is an operator on H for which $(T_x, x) = 0$ for all x, then show that T = 0.
8. If T is an operator on H. Then prove that T* T = I if $(T_x, T_y) = (x, y)$ for all x and y.

Section - B

Answer **all** questions. All questions carry **equal** marks. **(4×18=72)**

9. Let M be a closed linear subspace of a normed linear space N. If the norm of a coset x + M in the quotient space N/M is defined by $\|x + M\| = \inf\{\|x + m\| : m \in M\}$ then prove that N/M is a normed linear space. Also prove that if N is a Banach space so is N/M.

OR

10. State and prove Hahn - Banach theorem.

11. State and prove open mapping theorem.

OR

12. i) State and prove closed graph theorem.

ii) Prove that a non empty subset x of a normed linear space N is bounded if and only if f(x) is a bounded set of numbers for each f in N*.

13. i) Let $\{e_1, e_2, ..., e_n\}$ be a finite orthonormal set in a Hilbert space H. If x is any vector in H, then prove that $\sum_{i=1}^{n} |(x, e_i)|^2 \leq \|x\|^2$ and $x - \sum_{i=1}^{n} (x, e_i) e_i \perp e_j$ for each j.

ii) State and prove Bessel's inequality.

OR

14. Let H be a Hilbert space and f be an arbitrary functional in H*. Then prove that there exists a unique vector y in H such that f(x) = (x,y) for every x in H.

15. i) Prove that an operator T on H. is unitary if and only if it is an isometric isomorphism of H onto itself

ii) If P is a projection on H with range M and null space N, then prove that $M \perp N$ if and only if P is self adjoint and in this case $N = M^{\perp}$.

OR

16. State and prove Spectral theorem.